Transient waves in 1D oscillating media: second-order homogenization and interface conditions

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Abstract

We consider waves propagating in one-dimensional elastic media, whose properties are periodically varying and which may be bonded by homogeneous (or other heterogeneous) media. The goals of the study was to (i) design a model capturing the dispersive effects of the microstructure and (ii) write adequate boundary and transmission conditions at the interfaces for other media to finally (iii) present a full model and a stable numerical scheme.

For long wavelengths, the asymptotic homogenization procedure provides a family of effective models, which can be interpreted as "strain" or "stress gradient"-type enrichements of the classical elastic model. We select one of these models to achieve a better agreement with the dispersion curve of a given material. Corresponding boundary and interface correctors are then established in the time-harmonic case. They are finally extended to the transient setting, along with a reformulation of the enriched wave equation as an hyperbolic system whose stability is proven. The final model efficiency is illustrated by numerical simulations.

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