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# Effective dynamics for low-amplitude transient elastic waves in a 1D periodic array of non-linear interfaces

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## Abstract

This presentation focuses on the time-domain propagation of elastic waves through a 1D periodic medium that contains non-linear imperfect interfaces, i.e. interfaces exhibiting a discontinuity in displacement and stress governed by a non-linear constitutive relation. The array considered is generated by a, possibly heterogeneous, cell repeated periodically and bonded by interfaces that are associated with transmission conditions of non-linear “spring-mass” type. More precisely, the imperfect interfaces are characterized by a linear dynamics but a non-linear elasticity law. The latter is not specified at first and only key theoretical assumptions are required. In this context, we investigate transient waves with both low-amplitude and long-wavelength, and aim at deriving homogenized models that describe their effective motion. To do so, the two-scale asymptotic homogenization method is deployed, up to the first-order. To begin, an effective model is obtained for the leading zeroth-order contribution to the microstructured wavefield. It amounts to a wave equation with a non-linear constitutive stress-strain relation that is inherited from the behavior of the imperfect interfaces at the microscale. The next first-order corrector term is then shown to be expressed in terms of a cell function and the solution of a linear elastic wave equation. Without further hypothesis, the constitutive relation and the source term of the latter depend non-linearly on the zeroth-order field, as does the cell function. Combining these zeroth- and first-order models leads to an approximation of both the macroscopic behavior of the microstructured wavefield and its small-scale fluctuations within the periodic array. Finally, particularizing for a prototypical non-linear interface law and in the cases of a homogeneous periodic cell and a bilaminated one, the behavior of the obtained models are then illustrated on a set of numerical examples and compared with full-field simulations. Both the influence of the dominant wavelength and of the wavefield amplitude are investigated numerically, as well as the characteristic features related to non-linear phenomena.

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